POWER LAW AND MHD BOUNDARY LAYER FLOW OVER A STRETCHING SHEET PROBLEMS WITH MATLAB SOLVER

A SYNOPSIS

Submitted

in the partial fulfillment of the requirements for the award of the degree of

DOCTOR OF PHILOSOPHY

in

MATHEMATICS

By

CH. VENKATESHWARLU

Under the guidance of

DR.MADDILETI PASUPULA

DEPARTMENT OF MATHEMATICS

MAHATMA GANDHI UNIVERSITY

NALGONDA, TELANGANA– 508 003

APRIL 2022

SYNOPSIS

The proposed study is an investigation on "Power law and MHD boundary layer flow over a stretching sheet problem with MATLAB solver". The thesis comprises of five chapters, which are as follows:

Chapter 1:	Preface
Chapter 2:	MHD Flow and heat transfer numerical solution over a stretching sheet
Chapter 3:	Chemical reaction, thermal stratification effects on MHD flow over a vertical stretching surface with heat source
Chapter 4:	Exponential power law fluid over a semi-infinite vertical plate with variable thermal conductivity in the presence of the induced magnetic field
Chapter 5:	Radiation impact of aligned magnetic field flow and heat transfer with Newtonian heating boundary condition and heat production

The projected work deals with the effects of the physical parameters on flow fields. The first chapter is an introduction to the subject highlighting its importance and a brief survey of available concerned literature. All the chapters are provided with much needed text supported by the simple and explanatory figures. A brief summary of the thesis is as follows:

Chapter 1: In this chapter, we have presented an elementary discussion of principles of heat and mass transfer, some basic terminology and definitions. The science of heat transfer seeks merely explains how heat energy may be transferred.

Heat transfer occurs in three different modes:

i. Conduction

ii. Convection

iii. Radiation

If a flow system consists of one or more elements whose concentration vary from one point to other point. The process of transfer of mass from higher concentration to lower concentration area is known as the mass transfer.

To understand the fluid dynamics problems, it is to be noted that the four fundamental equations of the Fluid Dynamics must be known. The equations are as follows

i Equation of state

ii Equation of Continuity

iii Equation of Motion

iv Equation of Energy

Also, non-dimensional physical parameters act as an important role in fluid mechanics because they give us an idea as to which terms are dominant in the governing equations so that only those terms are retained in attempting to find an approximate solution to the governing equations

like Magnetic field(M), Prandtl number (Pr), Grashof number (Gr), Mass Grashof number(Gc), dimensionless exponential index(n),heat source(Q), Chemical reaction parameter(Kr), Eckart number (Ec), Schmidt number (Sc), Permeability of porous media(K), etc.

Chapter 2: MHD Flow and heat transfer numerical solution over a stretching sheet

In this chapter, we have analyzed the effects of chemical reaction and inclined magnetic field on unsteady magneto-hydrodynamic heat and mass transfer flow past a semi infinite moving vertical porous plate in the presence of viscous dissipation.

The governing equations to the problem are as follows

$$f''' + ff'' = f'\left(f' + (Ha)^2\right) - \lambda\left(\lambda + (Ha)^2\right) \quad \ldots (2.6)$$

$$\theta'' + \Pr\left(Ec\left(\left(f''\right)^2 + \left(f'.Ha\right)^2\right) + \theta' f\right) = 0 \quad \ldots (2.7)$$

Equation (2.4) transformed to the following form:

$$\begin{aligned} &f = 0, f' = 1, \theta = 1 \quad : \xi = 0 \\ &f' = \lambda, \theta = 0 \qquad\quad : \xi = \infty \end{aligned} \quad \ldots (2.8)$$

Where, $Ha = H_o \mu_e \sqrt{\frac{\sigma_e}{\rho c}}$ (Hartmann number), $\Pr = \frac{\mu C_p}{k}$ (Prandtl number), $Ec = \frac{u_w^2}{C_p (T_w - T_\infty)}$ (Eckert number) and $\lambda = \frac{a}{c}$ (Velocity Parameter).

An assessment is taken of a viscous, incompressible, electrically conductive fluid near a stagnation point of stretching sheet with its own plane in the existence of a magnetic field for the steady 2D, laminar boundary layer flow. The governing flow equations are changed into a nonlinear ODEs by substituting the suitable similarity variables. The numerical solutions and effects of the flow parameters of the present study obtained by MATLAB inbuilt solver and presented through graphically.

The numerical solution of the heat transfers and MHD flow over a stretching sheet in the presence of the magnetic field. The numerical solution obtained with MATLAB inbuilt programmable solver bvp5c for the governing equations and four the following final conclusions

Conclusions:

- Both the Hartmann and Prandtl numbers are increasing, which results in a drop in temperature curves.
- Increasing the Eckert number reduces the result in the temperature curves whereas increasing the Hartmann number raises the temperature curves.
- The temperature curves drop for lower values of velocity parameter, whereas raising the Hartmann number increases the temperature curves for greater values of velocity parameter.
- There is a decrease in velocity curves as the Hartmann number goes up.
- With Eckert number and decreasing Hartmann parameter, the temperature distribution increases.
- Velocity decreases with an increase in the Prandtl number and angle of inclination.

This chapter had been published in JOURNAL OF CRITICAL REVIEWS – Volume,7(18) - December 2016, pp. 1329-1335, ISSN: 2394-5125.

Chapter 3: *Chemical reaction, thermal stratification effects on MHD flow over a vertical stretching surface with heat source*

The aim of this chapter is to An investigation is carried out on a vertically stretching surface while taking into account the heat stratification and chemical reactions where a magnetic field is applied transverse to the flow direction. Governing equations in numerical methods are used to arrive at a solution for the flow problem. With the help of similarity transformations non- linear PDEs are converted into non-linear ODE. With the help of Matlab bvp4c package numerical solution are obtained for various values of dimensionless specifications.

The governing equations to the problem are

Using the similarity and dimensionless variables, Eqs.(3.2) and (3.4) reduce to the subsequent form:

$$f''' + ff'' + Re_x Gr_x \theta + Re_x Gr_x \phi - (f')^2 - \frac{M^2}{Re_x} f' = 0 \quad \ldots (3.6)$$

$$\theta'' - Pr\ Pr\ f' \left(\theta + \frac{n}{(1-n)}\right) + Prf\theta' - SPr\theta = 0 \quad \ldots (3.7)$$

$$\phi'' - Sc(\phi\gamma Re_x + f'\phi) + Scf\phi' = 0 \quad \ldots (3.8)$$

The boundary constraints become in the resulting form:

$$f = 0, f' = 1, \theta = 0, \phi = 1, \quad at\ \eta \to 0$$

$$f' \to 0, \theta \to 0,\ \phi \to 0 \qquad as\ \eta \to 0 \quad \ldots (3.9)$$

The effects of different parameters on the flow have been discussed with the help of graphs.

Conclusions:

The built-in solver method in MATLAB was successfully used to conduct a comparative analysis of the effects of chemical reaction and thermal stratification on MHD flow across a vertical stretched surface with a heat source. Under various physical conditions, the behaviour of dimensionless velocity and temperature profiles is discovered. The following are the study's major findings summarised and itemised:

- Thermal stratification and reactions of chemical nature are unchanged fluid temperature and fluid concentration are directly proportional while fluid velocity is inversely proportional to magnetic specification.
- When the magnetic field kept uniform and chemical reaction, specifications kept constant, the fluid velocity and the fluid temperature is inversely proportional to thermal stratification specification while the fluid concentration is directly proportional.
- The velocity near the plate first increased and then decreases when evaluated.
- As magnetic field increased, it also increases the temperature at a slow pace.

This work had been published in IOP Conference Series: Materials Science and Engineering, 993 (2020) 012113, doi:10.1088/1757-899X/993/1/012113

Chapter 4: *Exponential power law fluid over a semi-infinite vertical plate with variable thermal conductivity in the presence of the induced magnetic field*

In this chapter the exponential power-law steady of electrically conducting fluid over a semi-infinite vertical plate with the impact of induced magnetic field and consideration of variable thermal conductivity. The governing PDEs are converted into nonlinear ordinary differential equations with the similarity transformations and the solutions are obtained through MATLAB inbuilt software package bvp4c. Different values of the governing parameters are graphically provided for velocity, temperature distributions and skin-frictions and the rare of heat transfer coefficients are presented tabular from.

The equations for continuity, momentum, energy and diffusion, which govern the flow field, are transformation the PDE's (4.1)-(4.3) are transformed to the non-linear ODE's are

$$f''' = 2(f')^2 - ff'' - 4\left(M\left(1-(f')^2\right)\right) \quad \ldots (4.7)$$

$$\theta'' = -\left(\Pr f\theta' + \beta \Pr(\theta')^2\right)/(1+\beta\theta) \quad \ldots (4.8)$$

The boundary conditions are

$$\begin{aligned} &f = f' = 0, \theta = 1 \; at \;\; \eta = 0; \\ &f' \to 1, \theta \to 0 \; as \quad \eta \to \infty \end{aligned} \quad \ldots (4.9)$$

The above equations are solved and the solutions are obtained through MATLAB inbuilt software package bvp4c. Different values of the governing parameters are graphically provided for velocity, temperature distributions and skin-frictions and the rare of heat transfer coefficients are presented tabular from.

Conclusions:

An investigation of a numerical study of electrically conducting fluid across a semi-infinite vertical plate through with exponential power law fluid with a uniform magnetic field in the presence of variable thermal conductivity is carried out. Important findings of the problems are cited below.

- Prandtl number does not show the effect on the velocity profile.
- With an increase of M the velocity and $f''(0)$ increases.
- Temperature and $\theta'(0)$ decreases with an increasing the values of $\Pr$ and M increases with and increasing the β values.
- The heat transfer rate diminishes with an increasing M and the result enhance $\Pr$ as increases.
- The velocity distribution depends on the magnetic parameter and hence the skin friction is constant with varied values of the other parameters.

This work had been published in International Journal of Advanced Science and Technology, Vol. 29, No. 7, (2020), pp. 1144-1150

Chapter 5: Radiation impact of aligned magnetic field flow and heat transfer with Newtonian heating boundary condition and heat production

The aim of this chapter is the radiation effect of an aligned magnetic field on the steady boundary layer flow and heat transfer across a stretching sheet with Newtonian heating(NH) boundary condition in the existence of heat generation. The corresponding partial differential equations(PDE's) have been transformed into ordinary differential equations with aid of relevant transformations. The simplified ordinary differential equation system was numerically solved using the built-in solver of bvp4c MATLAB. The effect of various factors on dimensionless velocity and temperature is investigated. Between the current and earlier limiting findings, a good agreement is observed.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \ldots (5.1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=v\left(\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma}{\rho}uB_0^2 sin^2\alpha \qquad \ldots (5.2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}+\frac{Q_0}{\rho C_p}(T-T_\infty) \qquad \ldots (5.3)$$

With the boundary conditions

$$u=u_w(x)=ax, v=0, \frac{\partial T}{\partial y}=-hT(NH)\ \ At\ y=0\ u\to 0, T\to T_\infty \qquad at\ y\to\infty\ \} \qquad \ldots (5.4)$$

Conclusions:

An investigation is made into the numerical solutions for the flow of an aligned magnetic field and heat transfer across a stretching sheet fluid with Newtonian heating.

- Increasing the aligned angle and magnetic field factors, which lowers fluid velocity and raises the temperature profile, respectively.
- As the Prandtl number rises, the temperature profile declines. Meanwhile, when the temperature profile rises with enhancing the conjugate parameter, heat generation, radiation, rises.
- The Nusselt number and skin friction decreases with an increase of aligned angle and magnetics parameter.
- The Nusselt number and skin friction enhances with an increase of conjugate parameter.

This chapter not yet published.

REFERENCES

1. Acharya. M., P.Singh. L, and Dash, G.C: Heat and mass transfer over an accelerating surface with heat source in presence of suction and blowing. Int. J. of Engineering Science, 37(2),1999, pp.189-211. doi.org/10.1016/S0020-7225 (98)00064-0.
2. Ahmed, N. and Das K.: MHD Mass transfer flow past a vertical porous plate embedded in a porous medium in a slip flow regime with thermal radiation and chemical reaction. Open Journal of Fluid Dynamics, 3, 2013, pp.230-239.
3. Akira Nakayama., and Hitoshi Koyama: Similarity solutions for buoyancy-induced flows over a non-isothermal curved surface in a thermally stratified porous medium" Applied Scientific Research 46, 1989, pp.309-322. DOI10.1007/BF01998548.
4. Bilal,S., Khalil Ur Rehman and Malik, M.Y.:Numerical investigation of thermally stratified Williamson fluid flow over a cylindrical surface via Keller box method. Results in Physics, 7, (2017) pp.690–696. http://dx.doi.org/10.1016/j.rinp.2017.01.032.
5. Cortell R.: Flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation/absorption and suction/blowing. Fluid Dyn, 37(4), 2005, pp.231–45.
6. Cortell, R.: Viscous flow and heat transfer over a nonlinearly stretching sheet," Appl. Math and Comp., 184(2),2007, pp. 864-873.
7. Elbashbeshy, E.M.A. (2000) Radiation Effect on Heat Transfer over a Stretching Surface. *Canadian Journal of Physics*,**78**, 1107-1112.
8. Fan, J., J.Shi, and X. Xu. : Similarity solution of free convective boundary-layer behaviour at a stretching surface. Heat Mass

Transf., **35**, 1999, pp.191-196. https://doi.org/10.1007/s002310050313.

9. Ferdows, M., Bangalee M.Z.I. and Liu, D.: Electrically conducting flow through exponential power law fluid with variable thermal conductivity. Int. J. of Applied Mechanics and Engineering, 2019, 24(3), pp.539-548. doi.org/10.2478/ijame-2019-0034.
10. Gupta., P.S and Gupta, A.S: Heat and mass transfer on stretching sheet with suction or blowing. Canadian Journal of Chemical Engineering, 55, 1977, pp.744-746.
11. Hasmawani Hashim, Muhammad Khairul Anuar Mohamed, Nazila Ishak, Norhafizah Md Sarif, and Mohd Zuki Salleh: Thermal radiation effect on MHD Stagnation Point flow of Williamson fluid over a stretching Surface" Journal of Physics: Conference Series 1366,1-12(2019) 012011.
12. Hayat, T., Qasim, M., Mesloub, S: MHD flow and heat transfer over permeable stretching sheet with slip conditions, Internat. J. Numer. Methods Fluids, 66, 2011, pp. 963–975.
13. Ibrahim, F. S., Elaiw, A. M., and Bakr, A. A.: Effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Communications in Nonlinear Science and Numerical Simulation, 13(6), pp. 1056–1066, 2008. doi.org/10.1016/j.cnsns.2006.09.007.
14. ImranChandarki :On Series Solution of MHD Flow over a Stretching Permeable Surface. Procedia Engineering,127, 2015, pp.1278-1284, https://doi.org/10.1016/j.proeng.2015.11.484.
15. Ioan Pop and Tsung-YenNa: Unsteady flow past a stretching sheet. Mechanics Research Communications, 23(4),1996, pp. 413-422, https://doi.org/10.1016/0093-6413(96)00040-7.

16. Ishak, A. Nazar R. and Pop I.: Boundary layer flow and heat transfer over an unsteady stretching vertical surface.Meccanica, 44, 2009, pp.369-375.
17. Jat R.N., and Santosh Chaudhary: MHD Flow and Heat Transfer over a Stretching Sheet. Applied Mathematical Sciences,3(26), 2009, pp.1285-1294.
18. Kandasamy,R., Periasamy, K. Sivagnana Prabhu.,K.K.: Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects. International Journal of Heat and Mass Transfer 48, 2005, pp. 4557–4561. doi:10.1016/j.ijheatmasstransfer.2005.05.006.
19. Khadrawi, A.F. and Al-Odat, M.O.: Transient MHD Free convection flow over a permeable vertical moving plate embedded in porous medium with uniform suction. International Journal of Heat and Technology, 23, 2005, pp.81-87.
20. Liao, S.J., & Pop, I.: Explicit analytic solution for similarity boundary layer equations. International Journal of Heat and Mass Transfer, 47(1), 2014, pp.75–85. DOI:10.1016/s0017-9310(03)00405-8.
21. Macha Madhu and Naikoti Kishan :Magneto hydrodynamic mixed convection Stagnation-point flow of a power-law Non-Newtonian Nanofluid towards a stretching surface with radiation and heat source/sink.Journal of Fluids, Vol.2015,|Article ID 634186,14 pages, doi.org/10.1155/2015/634186.
22. Magyari E. and Keller B.: Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. Journal of Physics D, 32(5),1999, pp.577–585.
23. Makinde,O.D., and Ogulu, A., :The effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field:

Chemical Engineering Communications, 195(12),2008, pp. 1575–1584. doi.org/10.1080/00986440802115549.

24. Metri, P.G., Metri P.G., and Abel S., Silvestrov S.: Heat transfer in MHD mixed convection viscoelastic fluid flow over a stretching sheet embedded in a porous medium with viscous dissipation and non-uniform heat source/sink, Procedia Eng. 157, (2016), 309-316.
25. Mukhopadhyay, S.:The effects of thermal radiation on unsteady mixed convection flow and heat transfer over a porous stretching surface in porous medium. *International Journal of Heat and Mass Transfer,2009,*52, 3261–3265.
26. Pal, D. and Mondal, H., Radiation effects on combined convection over a vertical flat plate embedded in a porous medium of variable porosity", Acta Mec., 44(2), 2009, pp. 133–144. Doi:10.1007/s11012-008-9156-0.
27. Prasad, V. R., R. Muthucumaraswamy and B. Vasu.: Radiation and Mass transfer effects on unsteady MHD free convection flow past a vertical porous plate embedded in porous medium: a numerical study., Int. J. of Appl. Math and Mech., 6(19), 2010,pp.1 – 21.
28. Rai Sajjad Saif, Tasawar Hayat, Rahmat Ellahi, Taseer Muhammad and Ahmed Alsaedi: Stagnation-point flow of second grade nanofluid towards a nonlinear stretching surface with variable thickness. Results in Physics 7, 2017, pp. 2821–2830. http://dx.doi.org/10.1016/j.rinp.2017.07.062.
29. Rama Subba Reddy Gorla and I. Sidawi,: Free convection on a vertical stretching surface with suction and blowing, *Appl. Sci. Res.* **52,** 1994, 247–257.
30. Rout, B.R., Parida, S.K. and Panda, S. :MHD heat and mass transfer of chemical reaction fluid flow over a moving vertical plate in presence of heat source with convective surface boundary condition.

International Journal of Che. Engineering, Article ID 296834, 2013, Pages.10. https://doi.org/10.1155/2013/296834.

31. Sakiadis BC: Boundary layer behavior on continuous solid surfaces: II. The boundary layer on a continuous flat surfaces. A I Ch E J. 7, 1961, pp.221-25. [2.4]
32. Sakiadis, B. C.,:Boundary-Layer Behaviour on Continuous Solid Surfaces. I. Boundary Layer Equations for Two-Dimensional and Axisymmetric Flow., AIChE J., 7(1),1961, pp.26-28. https://doi.org/10.1002/aic.690070108.
33. Shankar Goud**,** B. and Narender, G. and Ranjit Kumar E.: Stagnation point flow through a porous medium towards stretching surface in the presence of heat generation", International Journal of Engineering and Advanced Technology, 9(1), 2019, pp-2646-2649.
34. Srinivasulu. T., Shankar Bandari and Chenna. Sumalatha: MHD stagnation point flow of Casson Nanofluid over a stretching sheet with effect of viscous dissipation. Global Journal of Pure and Applied Mathematics, 13(8), 2017, pp. 4229-4244.
35. Tripathy R, Dash G, Mishra S and Baag S.,:Chemical reaction effect on MHD free convective surface over a moving vertical plate through porous medium. Alex. Engg. Journal, 54(3), 2015, pp 673-79. http://dx.doi.org/10.1016/j.aej.2015.04.012.
36. Vajravelu K, Roper T.: Flow and heat transfer in a second grade fluid over a stretching sheet. Int. Journal of Non-Linear Mech.,34(6),1999,pp.1031-1036. https://doi.org/10.1016/S0020-7462(98)00073-0.
37. Venkateshwarlu. CH., Maddileti Pasupula. Dr., Srinivasulu Thadakamalla: Exponential power law fluid over a semi-infinite vertical plate with variable thermal conductivity in the presence of the induced magnetic field. Int. J. Adv Sci. and Tech.,20(7), 2020, pp.1144-1150.

38. Venkateshwarlu., CH., Maddileti Pasupula, and Srinivasulu Thadakamalla.: Exponential power law fluid over a semi-infinite vertical plate with variable thermal conductivity in the presence of the induced magnetic field. Int. J. Adv Sci. and Tech.,20(7), 2020, pp.1144-1150.

39. Yasin Abdela, Bandari Shankar, Srinivasulu T.:Numerical Solution of Stagnation Point Flow Of Nano Fluid Due to an Inclined Stretching Sheet. International Journal of Computational Engineering Research, 8(2), 2018, pp.21-34.

40. Zaib, K. Bhattacharyya, M. Khalid, and S. Shafie.: Thermal radiation effect on a mixed convection flow and heat transfer of the Williamson fluid past an exponentially shrinking permeable sheet with a convective boundary condition. Journal of Applied Mechanics and Technical Physics, 2017, 58(3), pp. 419–424.